Do large abelian monopole loops survive the continuum limit?

M. Grady[a]

[a]Department of Physics, SUNY Fredonia, Fredonia NY 14063 USA

An analysis of the monopole loop length distribution is performed in Wilson-action SU(2) lattice gauge theory. A pure power law in the inverse length is found, at least for loops of length, l, less than the linear lattice size N. This power shows a definite β dependence, passing 5 around $\beta = 2.9$, and appears to have very little finite lattice size dependence. It is shown that when this power exceeds 5, no loops any finite fraction of the lattice size will survive the infinite lattice limit. This is true for any reasonable size distribution for loops larger than N. The apparent lack of finite size dependence in this quantity would seem to indicate that abelian monopole loops large enough to cause confinement do not survive the continuum limit. Indeed they are absent for all $\beta > 2.9$.

1. INTRODUCTION

Much evidence has been presented that abelian monopoles extracted from SU(2) lattice gauge theory in the maximal abelian gauge are related to the confinement mechanism[1]. The monopoles appear to carry the entire SU(2) string tension. Long loops of monopole current at least as large as the lattice appear to be necessary for confinement. Evidence has also been presented that these monopoles survive the continuum limit[2]. It is important to ask whether these surviving monopoles exist in large loops capable of causing confinement, or only in small loops which are of zero physical size in the continuum.

2. MONOPOLE LOOP DISTRIBUTION

SU(2) Wilson-action lattices are transformed to maximal abelian gauge and abelian monopoles extracted using the DeGrand-Toussaint procedure[3]. The number of loops of each length are tabulated. It is found that small loops are more common than large loops. At larger β this effect is strongly enhanced. Define $p(l)$ to be the probability, normalized per lattice site, of a loop of size l occurring on a lattice. The quantity $N^4 p(l)$ gives the average number of loops of size l occurring on an N^4 lattice. In Fig. 1 $\log_{10} P(l)$ is plotted vs. $\log_{10}(l)$. It is seen that in all cases a power law is followed for smaller loops, up to a point somewhat beyond $l = N$. Beyond this there is a bulge of excess probability, followed by a sharp drop. This bulge is easily understood as a finite size effect due to the periodic boundary condition. Would-be large loops can reconnect through the boundary; thus one obtains an excess of mid-size loops ($l > N$) but a deficit of very large loops. This bulge is not so apparent for larger β. At any rate, the loop distribution certainly appears to follow a pure power law for $l \leq N$. Similar results were obtained in [4]. This power appears

Figure 1. Log-log plots of loop probability vs. loop size. Lines (from upper to lower) are fits to 12^4 data for $\beta = 2.6$,2.7,2.8,2.9, and 3.0

Figure 2. The power, q vs. β.

to have hardly any finite lattice size dependence, with the 12^4 data lying very close to the 20^4. The main difference in the data is a slight shift in the intercept. Indeed, since the power is already established by loops of length 6 and 8, it is hard to imagine how it could differ on very large lattices from what is seen here for the 20^4 lattice, especially considering how little difference there is between the 12^4 and 20^4 data. It seems unlikely that these small loops would be sensitive to the overall lattice size. Thus the power will likely be the same on very large lattices as observed here for the 20^4 lattice. It should be noted that minimal loops of size 4 do not fall on the power law line and are excluded from fits, and, with good statistics one can also see that the size 6 loops fall very slightly below the power trend. Taking the power law to be $p(l) \propto l^{-q}$, the power q obtained from linear fits is plotted vs. β in Fig. 2. A strong β dependence is apparent. This is in contradistinction to Ref. [4], where it was assumed there is no β dependence in this quantity. Above $\beta = 2.85$, $q > 5$, the significance of which is explained below.

Consider the probability of finding a loop of any size between N/b and N on an N^4 lattice, where b is fixed. For large N and $b << N$, this is given by $N^4 I(N,b)$ where

$$I(N,b) = \int_{N/b}^{N} p(l)dl. \qquad (1)$$

Here a sum over lengths has been replaced by an integral since N and N/b are large. Taking $p(l) \propto l^{-q}$, $N^4 I(N,b) \propto N^{5-q}$. Thus for $q > 5$, the probability of any loop in the size range N/b to N existing on an N^4 lattice for any fixed b vanishes as $N \to \infty$. If there are no loops in this range, it seems unlikely that larger loops could occur. Clearly, if the power law continues they do not. However, even if the loop distribution falls more slowly for $l > N$, such loops will not occur in the infinite lattice so long as the loop distribution continues to fall by a power ≥ 1. It would be nearly impossible for it to fall more slowly than this. For instance, assuming the distribution follows a different power law for $l > N$ (for which there is no evidence) it would look like $p(l) \propto N^{-(q-q')} l^{-q'}$ for $l > N$, where q is the power for $l < N$. The probability for having a loop of size $l > N$ on a lattice is $N^4 \int_N^{8N^4} p(l)dl$. It is easy to see that for $q > 5$ and $q' \geq 1$ this quantity vanishes for $N \to \infty$. Thus, somewhat paradoxically, the probability of large loops on large lattices is controlled by the probability distribution for $l < N$, where its behavior appears to be a pure power law, the power for which is established in turn by the behavior of very small loops.

To sum, what has been shown is that if the power, q, of loop probability falloff with loop length is larger than 5 in the region $l < N$, and larger than unity for $l > N$, then no loops any finite fraction of the lattice size exist in the infinite lattice limit. Previously it was seen from Fig. 2 that $q > 5$ for all $\beta \geq 2.9$. The apparent lack of dependence on lattice size for q means that this result should continue to hold for a very large and even infinite lattice. One is therefore led to the conclusion that monopole loops large enough to cause confinement do not survive the continuum ($\beta \to \infty$) limit. Therefore, if SU(2) lattice gauge theory confines in this limit, then confinement is not due to abelian monopole loops in the continuum. Another possibility is to accept that abelian monopole loops do cause confinement, in which case one is led to the conclusion that the continuum limit of SU(2) lattice gauge theory is not confined. Needless to say, either conclusion is

highly unconventional.

The monopole loops that do possibly survive the continuum limit are all of zero physical size, so are unlikely to affect the continuum theory in any way. This can be seen by taking a large but finite universe. Then the continuum limit can be taken by taking the lattice spacing $a \to 0$ and $N \to \infty$ together, keeping Na fixed at the universe size. Any physical size, such as that of a hadron, is a finite fraction of the universe size, Na/b, where b is finite. It was found above that no loops any finite fraction of the lattice size survive the continuum limit. The largest loop on a typical lattice must scale slower than N. Since a scales as $1/N$, such objects shrink to zero physical size in the continuum limit.

This suggests another way to analyze the data. One can measure the average largest loop (ALL) as a function of N and β. It is found that for $\beta < 2.9$, the ALL grows faster than N as N increases. Thus if wrapping loops are not present on a small lattice at such β they will eventually become common for large enough N. This is one way to understand why small lattices are deconfined and large lattices confine at the same β, and also why the critical beta of the deconfinement transition depends so much on N (see also [5]). However, for $\beta > 2.9$ it is found that the ALL scales *slower* than N in going from a 12^4 to a 20^4 lattice. Specifically, at $\beta = 3.0$, $\mathrm{ALL}/N = 0.463 \pm 0.004$ on the 12^4 and $\mathrm{ALL}/N = 0.418 \pm 0.005$ on the 20^4 lattice (errors are from binned fluctuations). Such behavior, of course, follows from the fact that $q > 5$ in the loop distribution. If this trend continues, then for these values of β the probability of a wrapping loop will *decrease* as N increases, and the confinement transition[1] will never occur, no matter how large N gets. The picture that emerges from this analysis is that the deconfinement transition does not, in fact continue to $\beta \to \infty$ as $N \to \infty$ but rather gets stuck around $\beta = 2.9$, and becomes a bulk transition on the infinite symmetric lattice, leaving the $\beta \to \infty$ continuum limit deconfined.

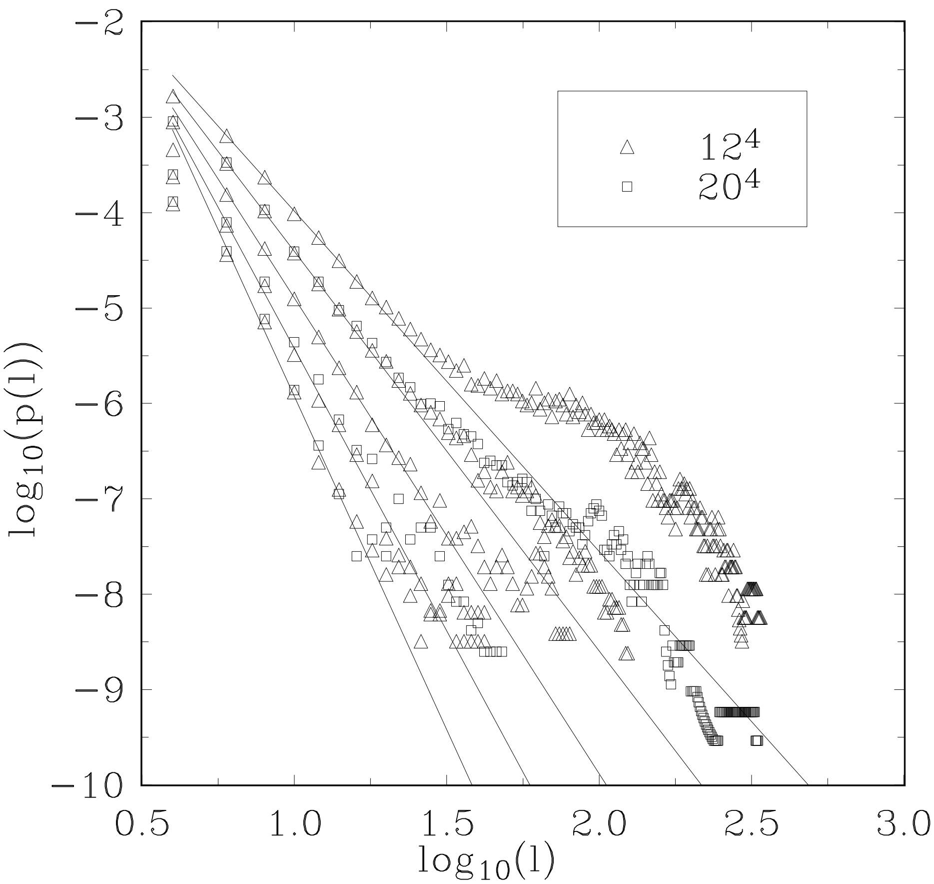
[1] It is, of course, not a true phase transition on a finite lattice.

3. SO(3)-Z_2 Monopoles

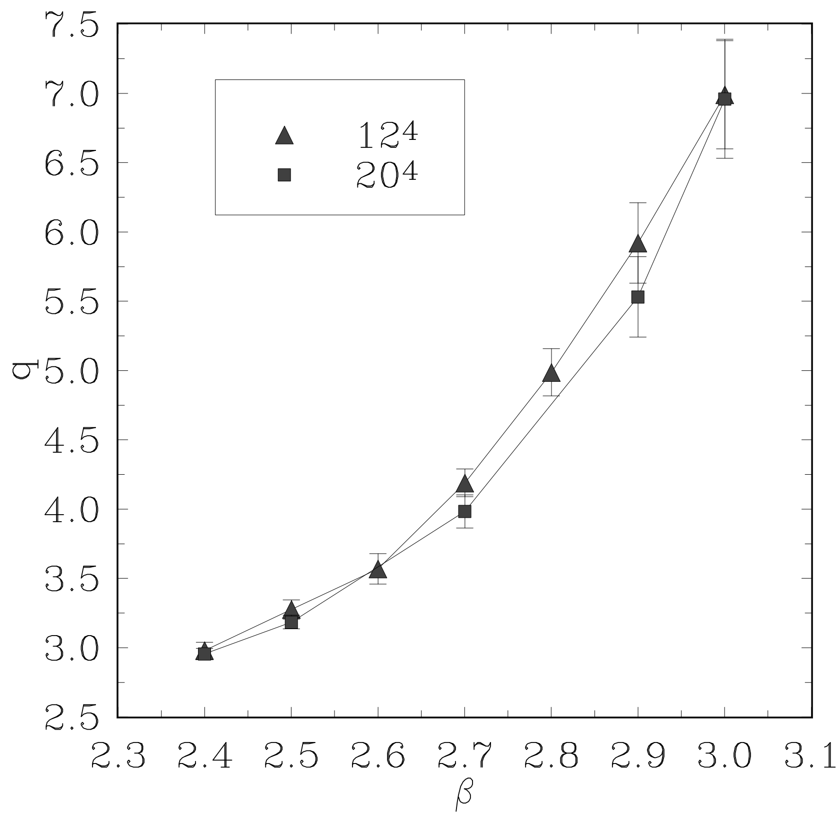
The possibility that the zero-temperature continuum limit is deconfined has been suggested before[6]. It is further strengthened by simulations which suppress a certain lattice artifact, the SO(3)-Z_2 monopole[7]. This monopole can be defined as a nontrivial realization of the lattice Bianchi identity[8], and carries both an SO(3) and Z_2 charge. It is an artefact because it is a local object of the scale of a single lattice spacing which requires large-angle plaquettes for support. Such objects cannot exist in the continuum, so an action which suppresses them should fall in the same universality class as the Wilson action. However, simulations with this action are deconfined for all β, despite the fact that this includes a region of rather strong renormalized coupling (from the average plaquette), indicating that hadronic scales have probably been reached. Analysis of the interquark potential for this action is underway, which should further elucidate this matter.

REFERENCES

1. A.S. Kronfeld, M.L. Laursen, G. Schierholz, and U.J. Weise, Phys. Lett. B **198**, 516 (1987); T. Suzuki and I. Yotsuyanagi, Phys. Rev. D **42**, 4257 (1990);
2. V.G. Bornyakov et. al., Phys. Lett. B, **261**, 116 (1991).
3. T.A. DeGrand and T. Toussaint, Phys. Rev. D **22**, 2478 (1980).
4. A. Hart and M. Teper, Nucl. Phys. B (Proc. Suppl.) **53**, 497 (1997).
5. M. Grady, SUNY-FRE-98-02, hep-lat/9802035 (unpublished).
6. M. Grady, Z. Phys C, **39**, 125 (1988); A. Patrascioiu, E. Seiler, and I.O. Stamatescu, Nuo. Cim. D **11**, 1165 (1989); R.V. Gavai, M. Grady, and M. Mathur, Nucl. Phys. B **423**, 123 (1994).
7. M. Grady, SUNY-FRE-98-09, hep-lat/9806024 (unpublished).
8. G. G. Batrouni, Nucl. Phys. B 208 (1982) 467; P. Skala, M. Faber, and M. Zach, Nucl. Phys B 494 (1997) 293.



-2
-3
-4
-5
-6
-7
-8
-9
-10
0.5
1.0
1.5
2.0
2.5
3.0
$\log_{10}(p(l))$
$\log_{10}(l)$
12^4
20^4



7.5
7.0
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
q
12^4
20^4
2.3
2.4
2.5
2.6
2.7
2.8
2.9
3.0
3.1
β